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Convection Diffusion Problems Solved by Fractional Variational Iteration Method

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Abstract

The paper considers the application of FVIM. The method is exploited explaining convection diffusion problems in different physical situations. These physical situations include energy, particles, etc., are transmitted inside the system owed to diffusion-convection.

Keywords: *Fractional Variational Iteration Method, Convection Diffusion problems, Kolmogorov-Petrovsly - Piskunov (KPP) equation*

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1. Introduction

In real world scenario, several important transport phenomena are explained using convection-diffusion problems. The diffusion and convection is frequent problem in physics which cause transport of a substance through a porous medium. These problems are applied in many investigations due to high importance in physical analysis and also numerical applicability of diffusion convection has high efficacy. The diffusion-convection explains heat flow, and other active physical quantities. The diffusion simply means change in concentration gradients. Bulk fluid motion, the flux of chemical species and convection are included. HPTM; namely homotopy perturbation transformation method for solving several convection-diffusion are presented by Gupta et al. [1]. This method has also been applied by Ghasemi and Kajani [2] for solving linear and nonlinear diffusion-convection problems. Another application has been presented by Liu and Zhao [3] Variational iteration method; VIM to solve one dimensional convection diffusion equation of unsteady. Further, Momani [4] has used Adomian decomposition method (ADM) for fractional convection diffusion equation of fractional type. One dimensional parabolic convection diffusion model is numerically solved using Bessel collocation method by Yuzbasi and Sahin [5].

ADM also used for numerical solution of different kinds of reaction diffusion convection by Wakil et al. [6]. Also, ADM used by Odibat, Jafari and Gejji [7,8] for solving wave equation i.e., diffusion equation of fractional order. Moreover, two dimensional diffusion equation and a fourth order explicit scheme of finite difference method was well explained by Hashim [9]. Wavelet method has also been used by Chen et al. [10] for space time fractional convectional diffusion equation with variable coefficients. FVIM proposed by J. H. He [11-19] has been applied in autonomous ordinary differential equations and produced results for infinite series because of no requirement of linearization and discretization.

2. Preliminaries

Definition 2.1. Consider a real function h (χ), χ > 0. It is called in space $C_\zeta, \zeta \epsilon R$ if ∍ a real no. b (> ζ), s.t. h (χ) = $\chi^b h_1(\chi)$, $h_1 \epsilon C[0,\infty]$. It is clear that $C_\zeta \subset C_\gamma$ if $\gamma \le \zeta$.

Definition 2.2. Consider a function h (χ), χ > 0. It is called in space $C_\zeta^m, m \,\epsilon\, \mathbb{N} \cup \{0\}$ if $h^{(m)} \epsilon\, C_\zeta$.

Definition 2.3. Left sided Caputo fractional derivative of h, $h \,\epsilon\, C_{-1}^m, m \in \mathbb{N} \cup \{0\}$,

$$D_t^\beta h(t) = \begin{cases} I^{m-\beta} h^{(m)}(t), m-1 < \beta < m, m \,\epsilon\, \mathbb{N}, \\ \frac{d^m}{dt^m} h(t),\ \beta = m, \end{cases}$$

a. $I_t^\zeta h(x,t) = \frac{1}{\Gamma\zeta}\int_0^t (t-s)^{\zeta-1} h(x,s)ds;\ \zeta, t > 0.$

b. $D_\tau^\nu V(x,\tau) = I_\tau^{m-\nu} \frac{\partial^m V(x,\tau)}{\partial t^m}, m-1 < \nu \le m.$

c. $D_t^\zeta I_t^\zeta h(t) = h(t)\ , m-1 < \zeta \le m, m \,\epsilon\, \mathbb{N}.$

d. $I_t^\zeta D_t^\zeta h(t) = h(t) - \sum_{k=1}^{m-1} h^k(0^+)\frac{t^k}{k!}, m-1 < \zeta \le m,, m \,\epsilon\, \mathbb{N}.$

e. $I^v t^\zeta = \frac{\Gamma(\zeta+1)}{\Gamma(v+\zeta+1)} t^{v+\zeta}.$

Definition 2.4. Laplace transform of Caputo fractional derivative is

$$L[D^\alpha g(t)] = p^\alpha F(p) - \sum_{k=0}^{n-1} p^{\alpha-k-1} g^{(k)}(0), n-1 < \alpha \le n.$$

Lemma[37]**.** If u and its partial derivatives are continuous, the fractional derivative $D_t^\alpha u(x,y,z,t)$ is bounded.

3. Basic plan of FVIM for nonlinear time-fractional system of differential equations

Consider the model described by

$$\frac{d^\alpha u}{dt^\alpha} = a_1 u + b_1 v(1-\epsilon v^2) + c_1, \quad (0 < \alpha \le 1) \tag{20}$$

with initial conditions u (0) = 0

A correction functional is formed for Eqs. (20) and (21) as:

$$u_{n+1}(t) = u_n + \frac{1}{\Gamma(1+\alpha)}\int_0^t \lambda\left(\frac{d^\alpha u_n}{d\tau^\alpha} - a_1\tilde{u}_n - b_1\tilde{v}_n\left(1-\epsilon\tilde{v}_n{}^2\right) - c_1\right)(d\tau)^\alpha, \tag{23}$$

By variational theory, λ must satisfy

$$\frac{d^\alpha \lambda}{dt^\alpha}|_{\tau=t} = 0$$

and $$1 + \lambda|_{\tau=t} = 0$$

Thus, we obtain $\lambda = -1$. Using in Eqs. (23) and (24), we have

$$u_{n+1}(t) = u_n - \frac{1}{\Gamma(1+\alpha)}\int_0^t \left(\frac{d^\alpha u_n}{d\tau^\alpha} + a_1 u_n - b_1 v_n(1-\epsilon v_n{}^2) - c_1\right)(d\tau)^\alpha,$$

Consecutive approximations u_n (t), n ≥ 0 can be built using Lagrange multiplier λ. The functions $\tilde{u}_n$ and $\tilde{v}_n$ are restricted variation i.e. $\delta\tilde{u}_n = 0$. Then we obtain sequences u_{n+1} (t) ,n ≥ 0.

The exact solution is gained as

$u(t) = lim_{n\to\infty} u_n(t),$

4. Numerical Approach

(4.1) Consider another problem: $\frac{d^\alpha u}{dt^\alpha} = \frac{d^2 u}{dt^2} - \frac{u}{4}$

with initial conditions: $u(x,0) = \frac{x}{2} + e^{\frac{-x}{2}}$

As before, we reach,

$u_{n+1}(t) = u_n + \frac{1}{\Gamma(1+\alpha)}\int_0^t \lambda\left(\frac{d^\alpha u_n}{d\tau^\alpha} - \frac{d^2 u_n}{dt^2} + \frac{u_n}{4}\right)(d\tau)^\alpha,$

We start with

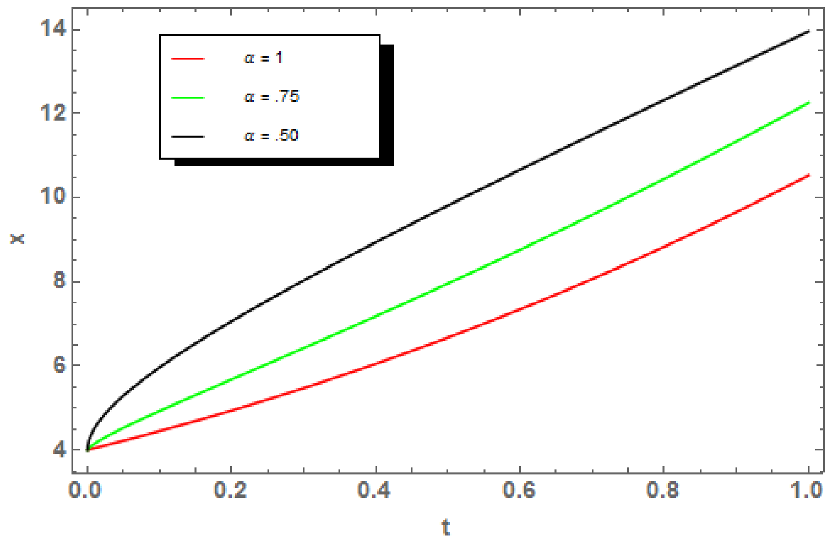
$u_0 = \frac{x}{2} + e^{\frac{-x}{2}}$

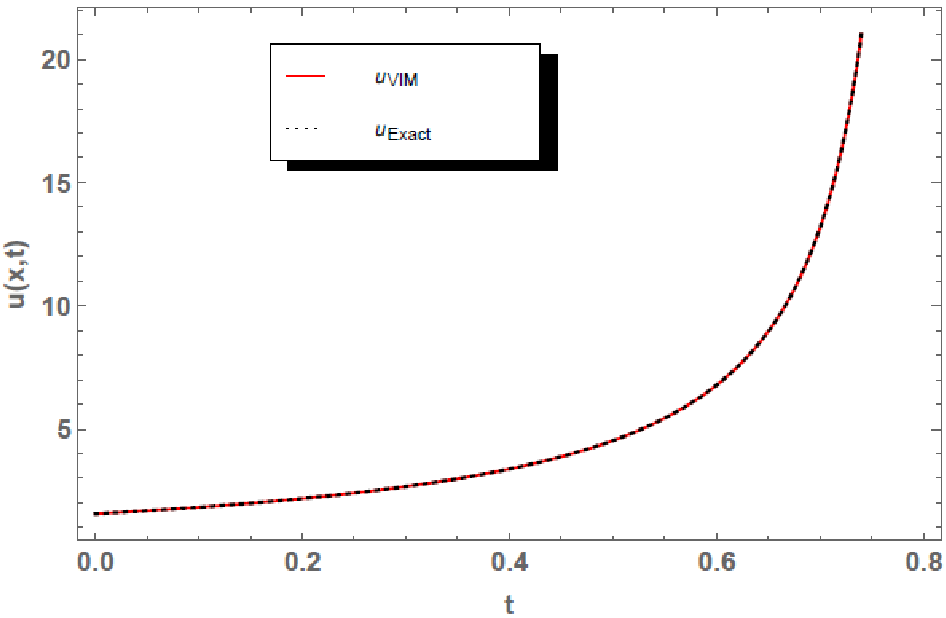
$$u_1 = \frac{1}{4}e^{-x/2}(4 + 2e^{x/2}x + \frac{t^\alpha(5 + e^{x/2}(-2+x))}{\Gamma(1+\alpha)})$$

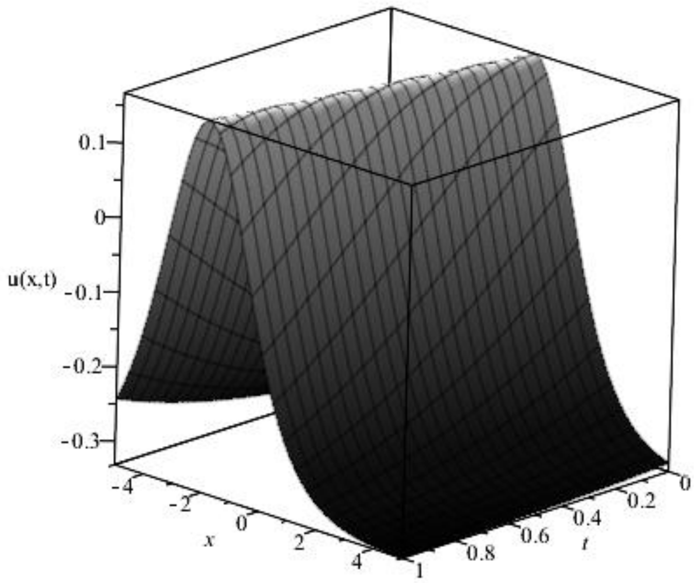
$$u_2 = \frac{1}{16}e^{-x/2}(8(2 + e^{x/2}x) + \frac{8t^\alpha(5 + e^{x/2}(-2+x))}{\Gamma(1+\alpha)} + \frac{t^{2\alpha}(25 + 2e^{x/2}(-4+x))}{\Gamma(1+2\alpha)})$$

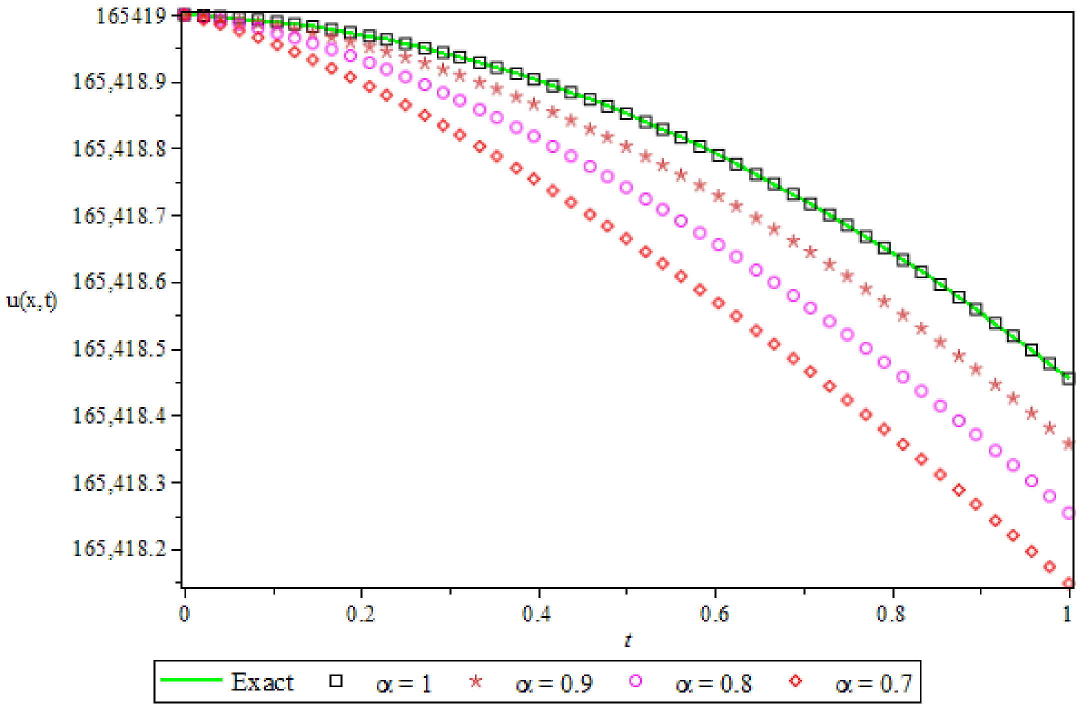
:

The solution is found to be $u(x, t) = e^{-x} + x\, e^{-t}$.









(4.2) Consider the Kolmogorov – Petrovsly - Piskunov (KPP) equation

$$\frac{d^{\alpha}u}{dt^{\alpha}} = \frac{d^2u}{dt^2} - 16u$$

Initial conditions: $u(x,0) = e^{-x-4}$

By iterative formula, $u_{n+1}(t) = u_n + \frac{1}{\Gamma(1+\alpha)}\int_0^t \lambda\left(\frac{d^{\alpha}u_n}{d\tau^{\alpha}} - \frac{d^2u_n}{dt^2} + 16u_n\right)(d\tau)^{\alpha},$

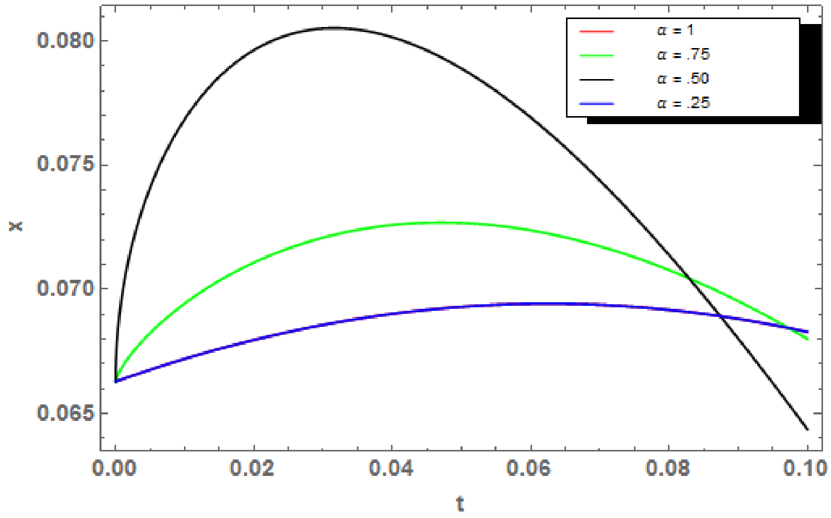
Starting with, $u_0 = e^{-x-4}$

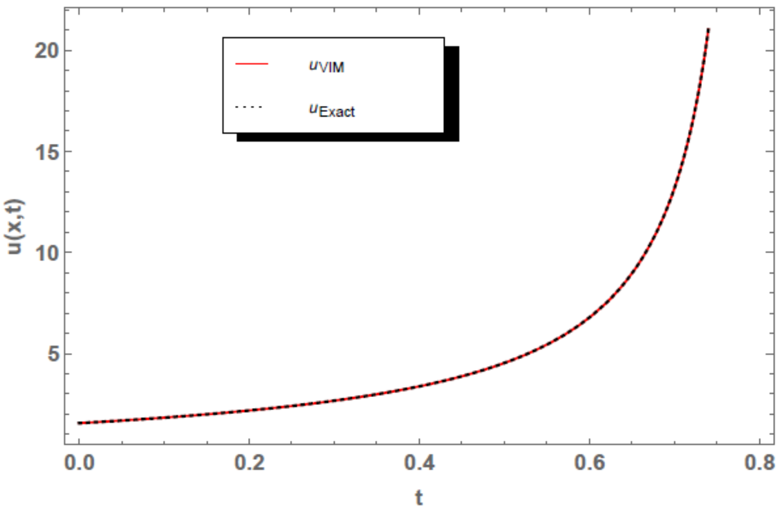
$$u_1 = e^{-4-x}\left(1 + \frac{t^{\alpha}(1-16t+\alpha)}{\Gamma(2+\alpha)}\right)$$

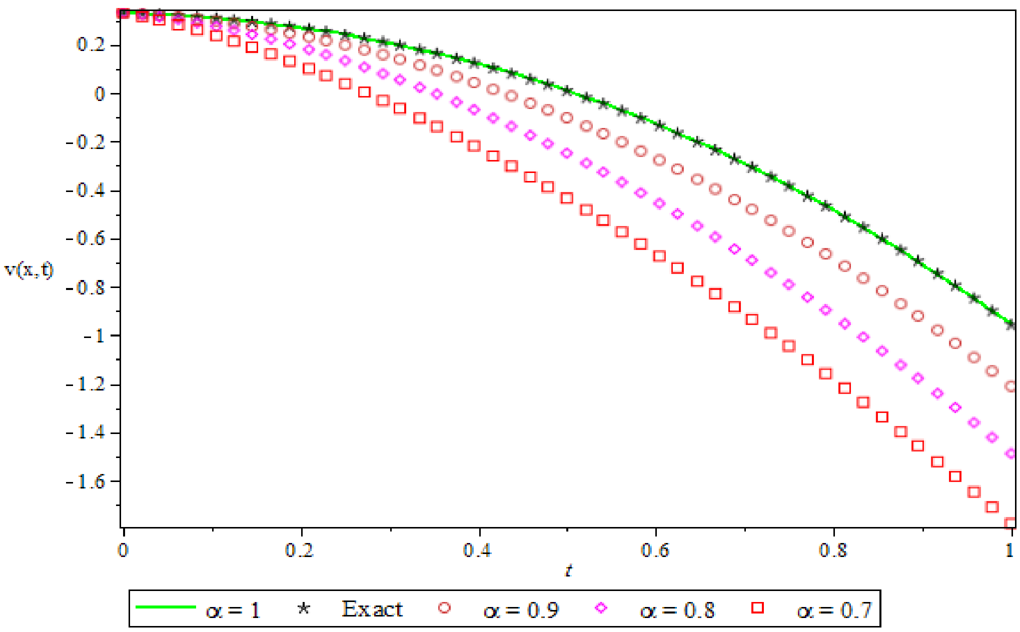
$$u_2 = e^{-4-x}\left(1 + 2t^{\alpha}\left(\frac{1-16t+\alpha}{\Gamma(2+\alpha)]} + \frac{t^{\alpha}(1+3\alpha+2(\alpha^2+64t^2(2+\alpha)-8t(1+\alpha)(2+\alpha)))}{\Gamma(3+2\alpha)}\right)\right)$$

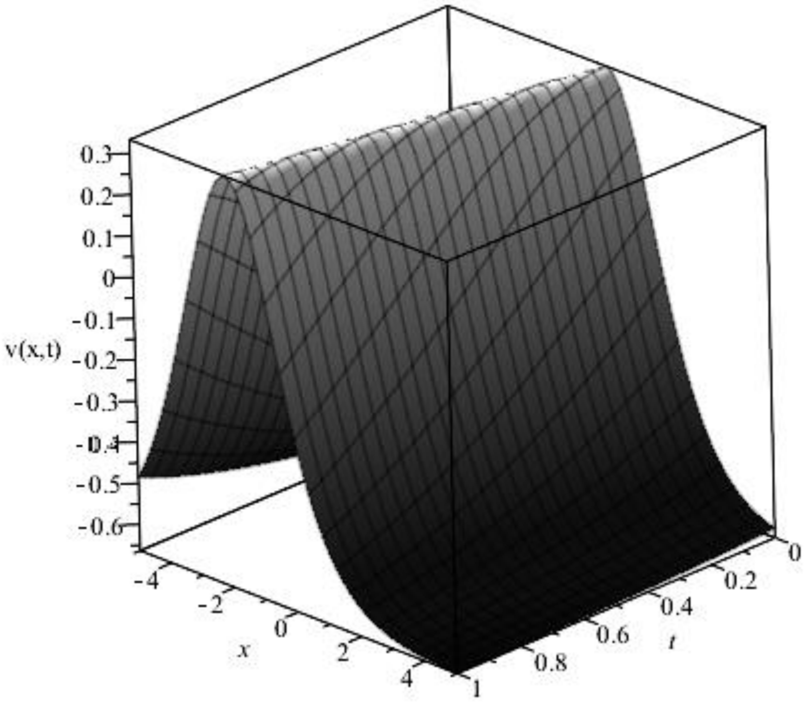
:

The result is obtained as u(x,t) = $e^{-x-4-8t^2+t}$.









5. Conclusion:

In this paper, linear and nonlinear convection-diffusion problems are solved by using application of VIM. It has been found that there is no need to get the Adomian polynomials and the method is accurate and effective to get the exact and analytical solution of the nonlinear problems. It skips the descretization error and inclusion of small parameter as require in HPM. Also, in this method the computation cost is minimum as compared to other existing methods.

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